

Hadron correlators at finite temperature

S. Shcheredin

(Bielefeld U.)

in collaboration with

S. Datta, F. Karsch, E. Laermann, S. Wissel

Outline

- Motivation
- Results:
 - Eigenvalues of a truncated perfect action
 - Spectral functions
 - Screening masses
 - Restoration of the chiral symmetry above T_C
 - Pion decay constants at finite T
- Conclusions and outlook

Motivation

- Existence of bound states in the deconfined phase
- Properties of bound states
- Effective chiral symmetry restoration at finite temperature (topological zero modes and their role)

Mesonic correlators at finite temperature

- Primary quantities are the thermal correlators

$$G_H(\tau, \vec{x}, T) = \left\langle J_H^\dagger(\tau, \vec{x}) J_H(0, \vec{0}) \right\rangle_T$$

$$J_H = \bar{\psi} \Gamma_H \psi, \quad \Gamma_H = 1, \gamma_5, \gamma_5 \gamma_\mu, \gamma_\mu$$

- Quark model: P=(-1)^{L+1}, C=(-1)^{L+S}

Γ	$^{2S+1}L_J$	J^{PC}	$u\bar{u}$	$c\bar{c}(n=1)$	$c\bar{c}(n=2)$	$b\bar{b}(n=1)$	$b\bar{b}(n=2)$
γ_5	1S_0	0^{-+}	π	η_c	η'_c	η_b	η'_b
γ_s	3S_1	1^{--}	ρ	J/ψ	ψ'	$\Upsilon(1S)$	$\Upsilon(2S)$
$\gamma_s \gamma_{s'}$	1P_1	1^{+-}	b_1	h_c		h_b	
1	3P_0	0^{++}	a_0	χ_{c0}		$\chi_{b0}(1P)$	$\chi_{b0}(2S)$
$\gamma_5 \gamma_s$	3P_1	1^{++}	a_1	χ_{c1}		$\chi_{b1}(1P)$	$\chi_{b1}(2P)$

Table taken from P. Petreczky [hep-lat/0606007](#)

- We are dealing only with light mesons
- L=1 states are expected to dissolve faster than L=0 states

Mesonic correlators at finite temperature

- Primary quantities are the thermal correlators

$$G_H(\tau, \vec{x}, T) = \left\langle J_H^\dagger(\tau, \vec{x}) J_H(0, \vec{0}) \right\rangle_T$$

$$J_H = \bar{\psi} \Gamma_H \psi, \quad \Gamma_H = 1, \gamma_5, \gamma_5 \gamma_\mu, \gamma_\mu$$

- Existence, masses and widths are encoded in spectral function

$$G_H(\tau, T) = \int_0^\infty d\omega \sigma_H(\omega, T) K(\tau, \omega, T)$$

- Thermal dilepton and photon production rate via the spectral function

$$\frac{d^4 W}{d^4 q} = \frac{5\alpha^2}{27\pi^3 \omega^2} \frac{1}{(e^{\frac{\omega}{T}} - 1)} \sigma_V(\omega, q, T)$$

- Method: ab initio non-perturbative computation of the spectral functions in the lattice QCD
- Lattice: renormalisation constants Z_H for point like currents

$$J_H^R(x) = Z_H J_H(x)$$

Maximum entropy method (MEM)

- Solving for σ is ill posed numerical problem on $V = L^3 \times T^{-1}$

$$G(\tau_i, T) = \sum_j \sigma(\omega_j, T) K(\omega_j, \tau_i, T) \Delta\omega, \quad \tau_i / a = 0, \dots, T^{-1} - 1, \quad \omega = 0, \dots, \omega_{\max}$$

$O(10)$ d.o.f. $\rightarrow O(1000)$ d.o.f.

- MEM: combine all known information using the Bayes theorem to reconstruct the spectral function (SF).
No functional assumption on the SF is made.

$$P(\sigma | DH) \sim P(D | \sigma H) P(\sigma | H)$$

– prior knowledge of H:

$$\sigma(\omega, T) > 0,$$

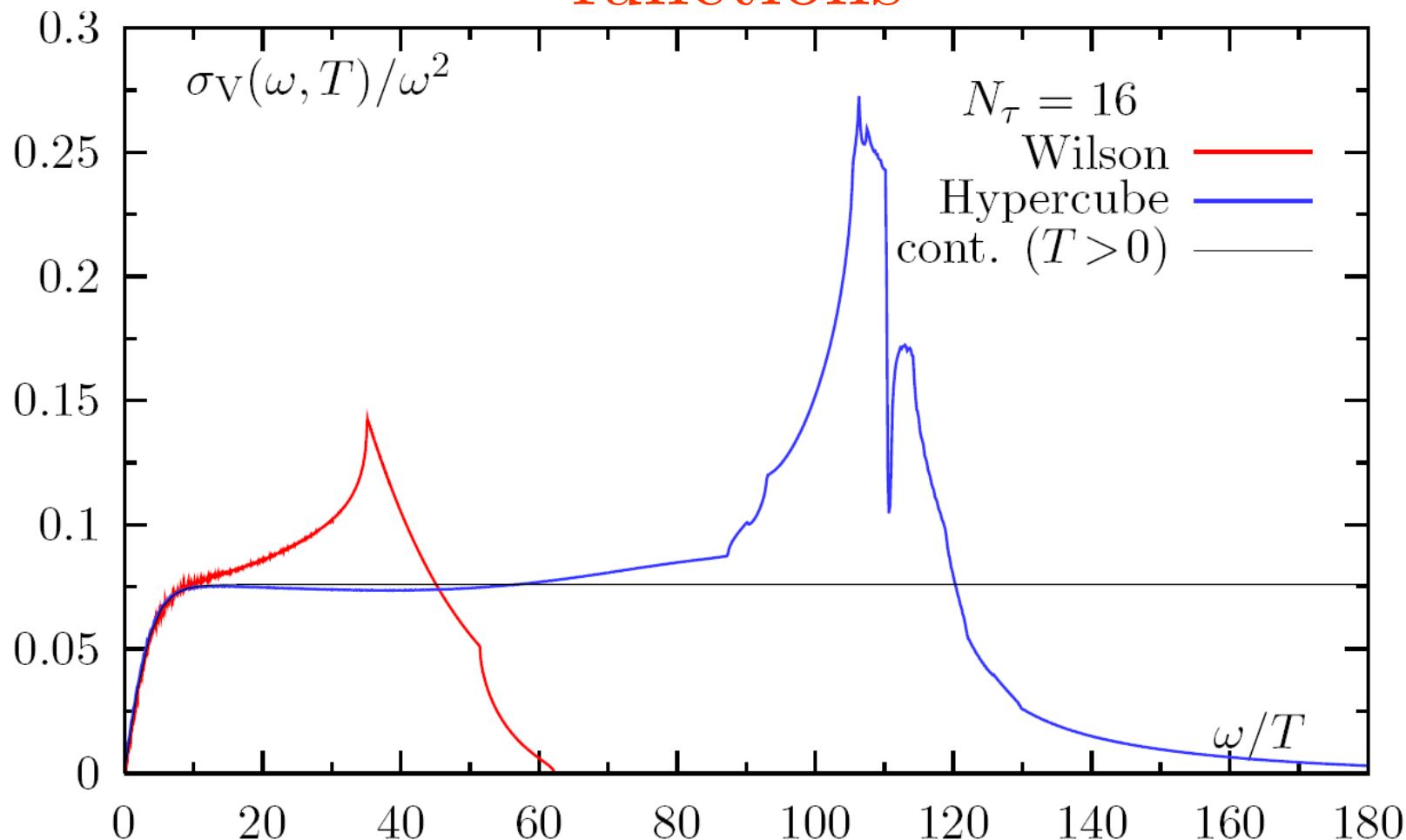
$\sigma(\omega \rightarrow \infty, T) \rightarrow$ free spectral function (default model)

M. Asakawa et al. 2001

Maximum entropy method (continued)

- at $T = 0$ or $T < T_c$:
MEM is successfully applied (sharp ground and excited states can be identified)
Y. Nakahara et al. 1999, T. Yamazaki et al., 2002
- at $T > T_c$: the states are expected to become broader whereas the “time” extension shrinks. Hence it is more difficult to analyze and disentangle the lattice artifacts.
- heavy quarks (charmonium) J/ψ , η_c ($L=0$) survive up to $1.6T_C$, χ_c melts at $1.1T_C$ ($L=1$)
P. Petreczky et al, M. Asakawa et al, H. Matsufuru et al., Dublin-Swansea
- Use of a truncated perfect action may become exceedingly efficient

Motivation: free lattice spectral functions



Longer plateau for the truncated perfect action

F. Karsch, E. Laermann, P. Petreczky, S. Stickan 2003

Truncated perfect fermions

- The holy grail is the quantum perfect action
 - physical quantities are free from any lattice artifacts at a rough lattice spacing
- Iterated Wilson renormalisation group transformation:
from the action of fields on a fine lattice (gauge fields, fermions) to coarse grained variables.

Iteration -> perfect action

- free fermion perfect action is known analytically
 - free of any lattice artifacts
 - obeys the Ginsparg-Wilson relation, hence chirally symmetric
- interacting case is much more complicated (we are using an approximation, hence the properties are somewhat distorted)

P. Hasenfratz, F. Niedermayer, 1994, W. Bietenholz, U.-J. Wiese, 1996

The lattice action

- the free perfect action is truncated to a hypercube

$$D_{\text{HF}}(x, y) = \sum_{\mu} \rho_{\mu}(x - y; \rho_1, \rho_2, \rho_3, \rho_4) \gamma_{\mu} + \lambda(x - y; \lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

$$\begin{aligned} \rho_{\mu}(x - y) &= \rho_1(\delta_{y,x+\hat{\mu}} - \delta_{y,x-\hat{\mu}}) + \sum_{\hat{\nu} \neq \hat{\mu}} \rho_2(\delta_{y,x+\hat{\mu}+\hat{\nu}} - \delta_{y,x-\hat{\mu}+\hat{\nu}}) \\ &\quad + \sum_{\substack{\hat{\nu} \neq \hat{\mu}, \hat{\rho} \\ \hat{\rho} \neq \hat{\mu}, \hat{\nu}}} \rho_3(\delta_{y,x+\hat{\mu}+\hat{\nu}+\hat{\rho}} - \delta_{y,x-\hat{\mu}+\hat{\nu}+\hat{\rho}}) \\ &\quad + \sum_{\substack{\hat{\nu} \neq \hat{\mu} \\ \hat{\rho} \neq \hat{\nu}, \hat{\sigma} \neq \hat{\rho}}} \rho_4(\delta_{y,x+\hat{\mu}+\hat{\nu}+\hat{\rho}+\hat{\sigma}} - \delta_{y,x-\hat{\mu}+\hat{\nu}+\hat{\rho}+\hat{\sigma}}), \end{aligned}$$

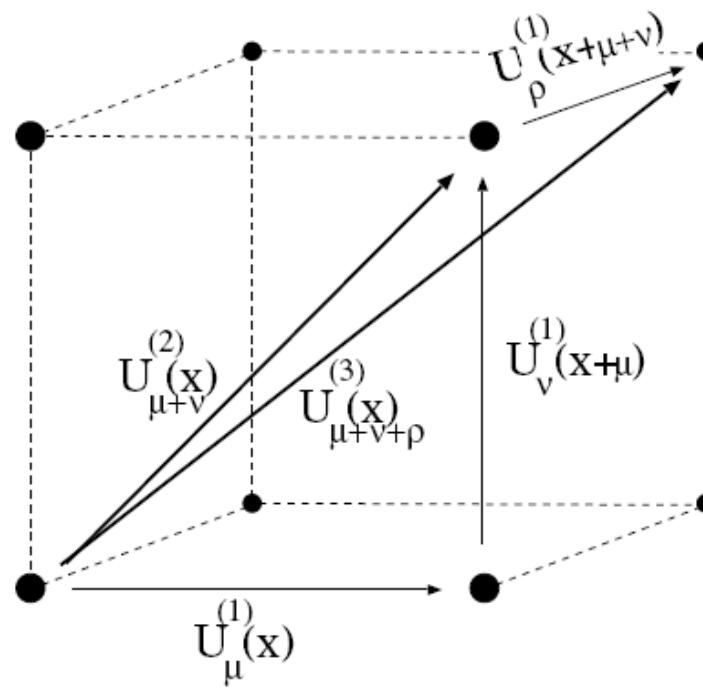
Couplings are fixed
in the free case from
“blocking from continuum”

$$\begin{aligned} \lambda(x - y) &= \lambda_0 \delta_{y,x} + \sum_{\mu} \lambda_1(\delta_{y,x+\hat{\mu}} + \delta_{y,x-\hat{\mu}}) + \sum_{\hat{\nu} \neq \hat{\mu}} \lambda_2(\delta_{y,x+\hat{\mu}+\hat{\nu}} + \delta_{y,x-\hat{\mu}+\hat{\nu}}) \\ &\quad + \sum_{\substack{\hat{\nu} \neq \hat{\mu} \\ \hat{\rho} \neq \hat{\mu}, \hat{\nu}}} \lambda_3(\delta_{y,x+\hat{\mu}+\hat{\nu}+\hat{\rho}} + \delta_{y,x-\hat{\mu}+\hat{\nu}+\hat{\rho}}) \\ &\quad + \sum_{\substack{\hat{\nu} \neq \hat{\mu} \\ \hat{\rho} \neq \hat{\nu}, \hat{\sigma} \neq \hat{\rho}}} \lambda_4(\delta_{y,x+\hat{\mu}+\hat{\nu}+\hat{\rho}+\hat{\sigma}} + \delta_{y,x-\hat{\mu}+\hat{\nu}+\hat{\rho}+\hat{\sigma}}). \end{aligned}$$

- gauging is introduced as an ansatz of hyperlinks, connecting sites on the unit hypercube

W. Bietenholz, U.-J. Wiese, 1996

$$\begin{aligned}
U_{\mu_1+\mu_2+\dots+\mu_d}^{(d)}(x) = & \frac{1}{d} \left[U_{\mu_1}^{(1)}(x) U_{\mu_2+\mu_3+\dots+\mu_d}^{(d-1)}(x + \hat{\mu}_1) \right. \\
& + U_{\mu_2}^{(1)}(x) U_{\mu_1+\mu_3+\dots+\mu_d}^{(d-1)}(x + \hat{\mu}_2) \\
& + \dots \\
& \left. + U_{\mu_d}^{(1)}(x) U_{\mu_1+\mu_2+\dots+\mu_{d-1}}^{(d-1)}(x + \hat{\mu}_d) \right]
\end{aligned}$$



The lattice action

- rescaling to fine tune the current quark mass of the hypercube operator to criticality

$$U_{x,\mu} \rightarrow uv U_{x,\mu} \quad \text{in } \rho_\mu(x-y, U)$$

$$U_{x,\mu} \rightarrow u U_{x,\mu} \quad \text{in } \lambda(x-y, U)$$

- corresponds to rescaling of the couplings

$$\lambda_0 \rightarrow \lambda_0 ,$$

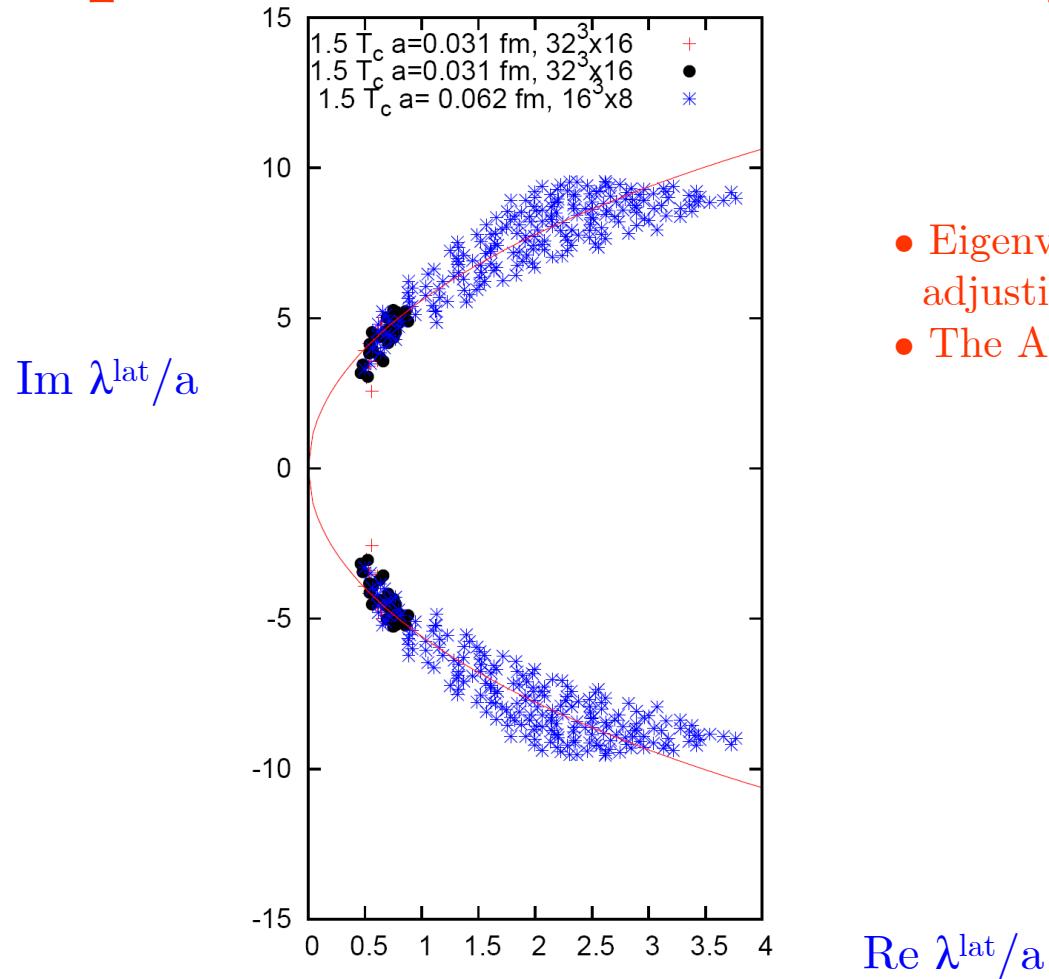
$$\lambda_1 \rightarrow u\lambda_1 , \quad \kappa_1 \rightarrow uv\kappa_1 ,$$

$$\lambda_2 \rightarrow u^2\lambda_2 , \quad \kappa_2 \rightarrow (uv)^2\kappa_2 ,$$

$$\lambda_3 \rightarrow u^3\lambda_3 , \quad \kappa_3 \rightarrow (uv)^3\kappa_3 ,$$

$$\lambda_4 \rightarrow u^4\lambda_4 , \quad \kappa_4 \rightarrow (uv)^4\kappa_4 ,$$

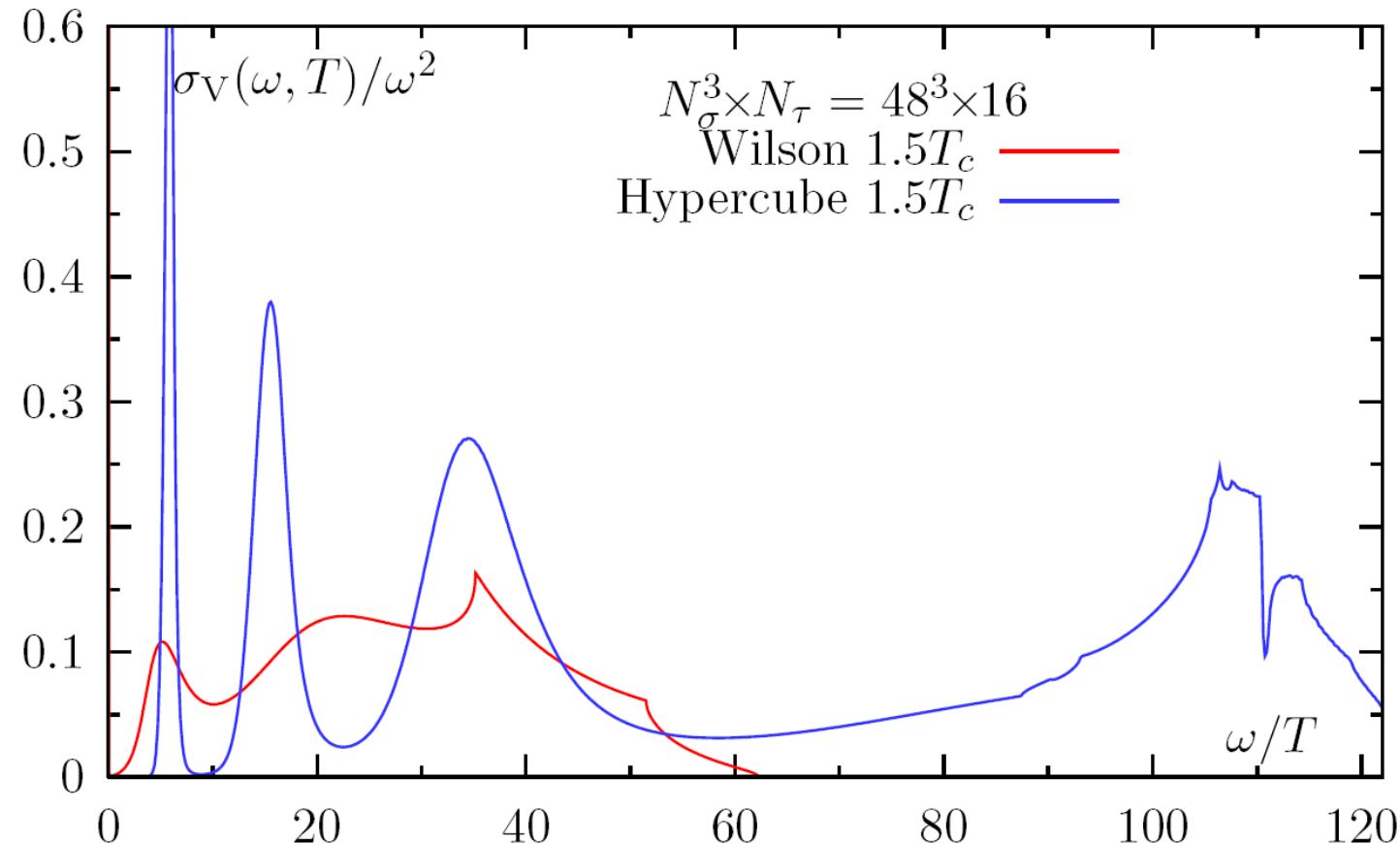
Eigenvalues of the hypercube operator at finite temperature QCD



- Eigenvalues are tuned to criticality by adjusting u and v .
- The AWI mass comes compatible with zero

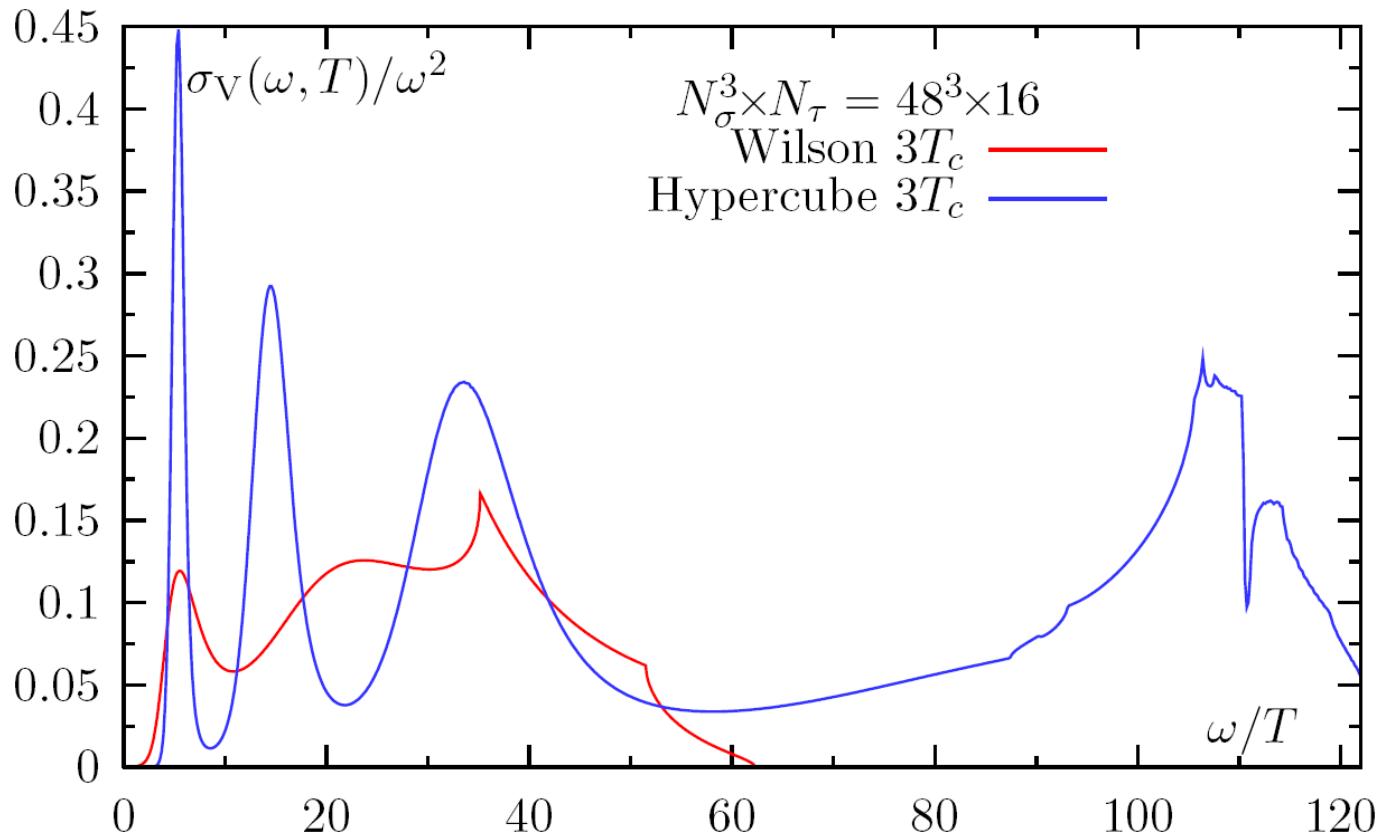
Very good scaling of the eigenvalue gap

Results: truncated perfect action vs. Wilson action



- The first peak agrees with the Wilson fermion result
- UV artifacts are pushed to much higher frequencies
- Caveat: number of the excited states strongly depends on the value of the Z_V and also can be affected by MEM artifacts

Results: truncated perfect action vs. Wilson action



- The first peak agrees with the Wilson fermion result
- UV artifacts are pushed to much higher frequencies
- Caveat: number of the excited states strongly depends on the value of the Z_V and also can be affected by MEM artifacts

Screening masses

- $G_H(x_3, T) = \int d\tau dx_1 dx_2 \left\langle J_H^\dagger(\tau, \vec{x}) J_H(0, \vec{0}) \right\rangle_T$

- Effective screening mass

$$m_{\text{screen}}(z) = -\frac{\partial \ln G_H(z)}{\partial z}$$

- The screening mass does not have to agree with the pole mass at finite temperature

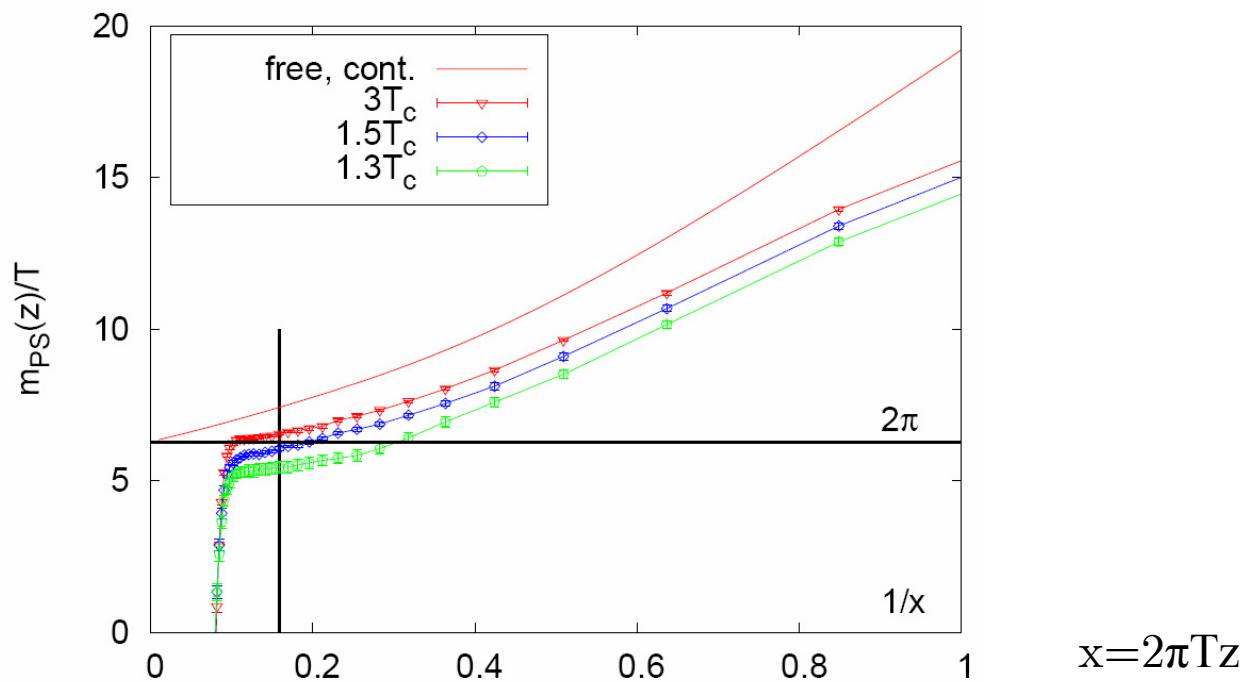
$$m_{\text{screen}}(T) = ? m_{\text{pole}}(T)$$

Effective screening mass

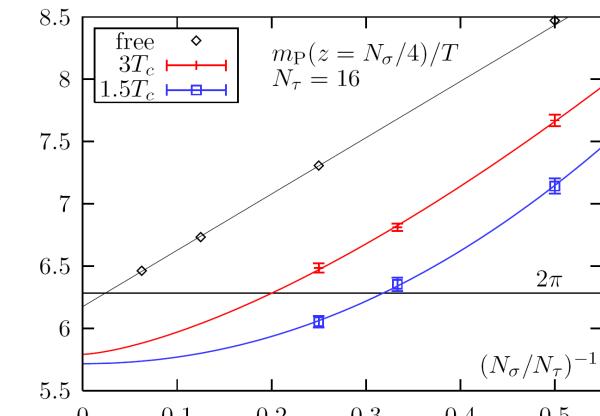
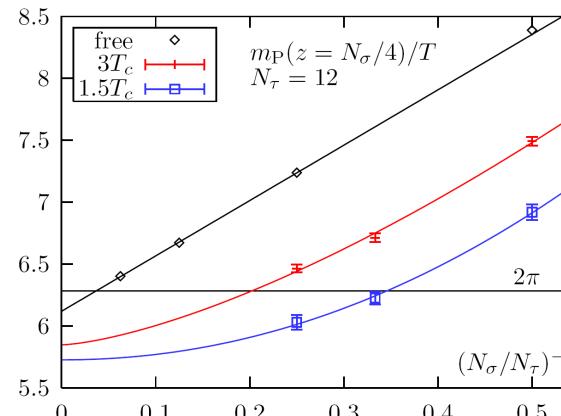
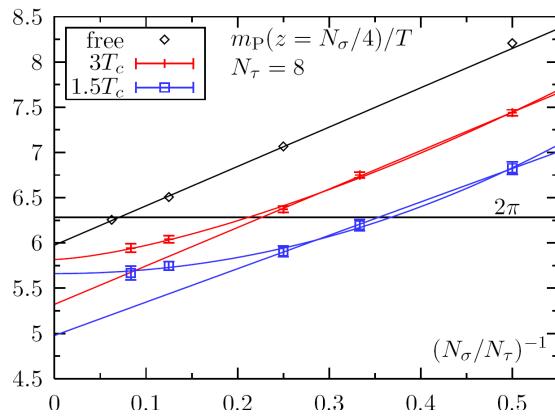
- $m_{\text{screen}}(z)$ goes asymptotically to $2\pi T$ at large z in the free case
- define at $z=L/4$ for the free and interacting case

$$m_{\text{screen}}(L/4) = 2\pi T \left(1 + \frac{2}{\pi L T} + \dots\right) \quad \text{free case}$$

- the asymptotics holds also at finite lattice spacing for the free case



Infinite volume extrapolation of the screening mass



p	PS	V
1.5 T _C	2.1 (4)	2.2 (5)
3 T _C	1.5 (2)	1.5 (2)

$$m_{\text{screen}}(a, L/4) = m_{\text{screen}}(a) \left(1 + \gamma \frac{1}{(L T)^p} + \dots \right)$$

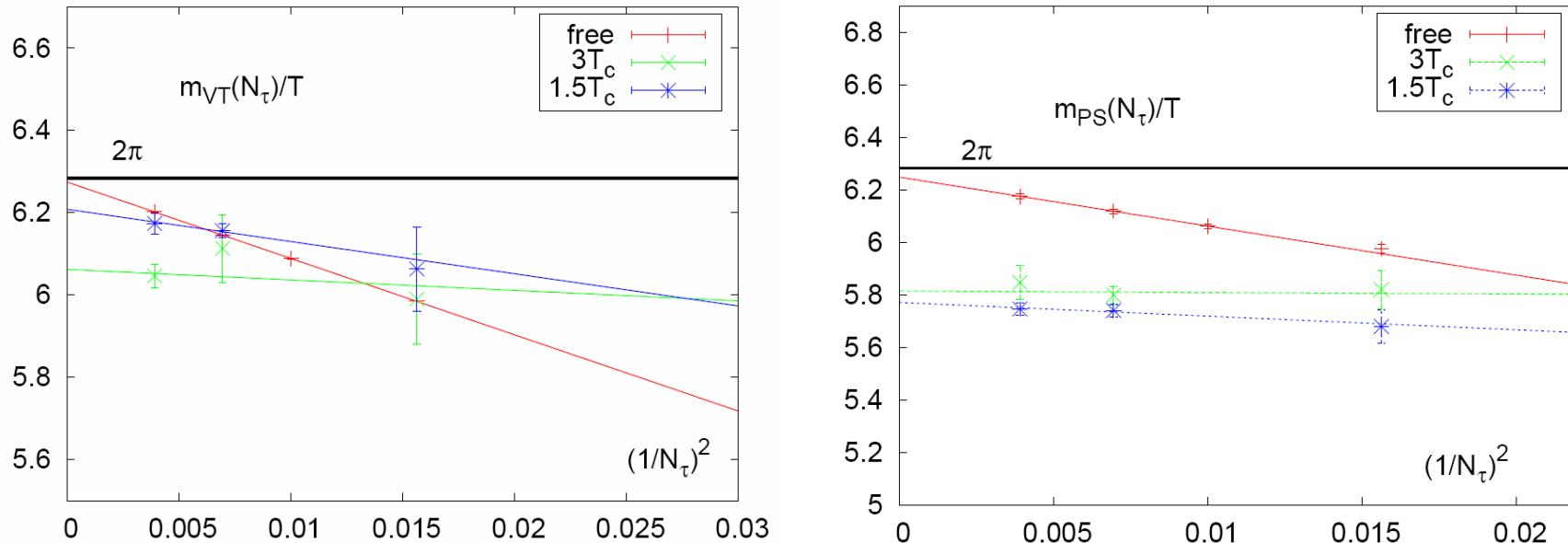
$$p = 3 \quad T < T_C$$

$$p = 1 \quad T \rightarrow \infty$$

Fit for $N_\tau = 8, N_\sigma / N_\tau = 2, 3, 4, 8, 12$

- The values approach p=1 as temperature is increased
- At $N_\tau=12, 16$ we use the values of p determined from $N_\tau=8$
- Data reveal substantial bending at large spatial extents

Continuum extrapolation of the screening mass

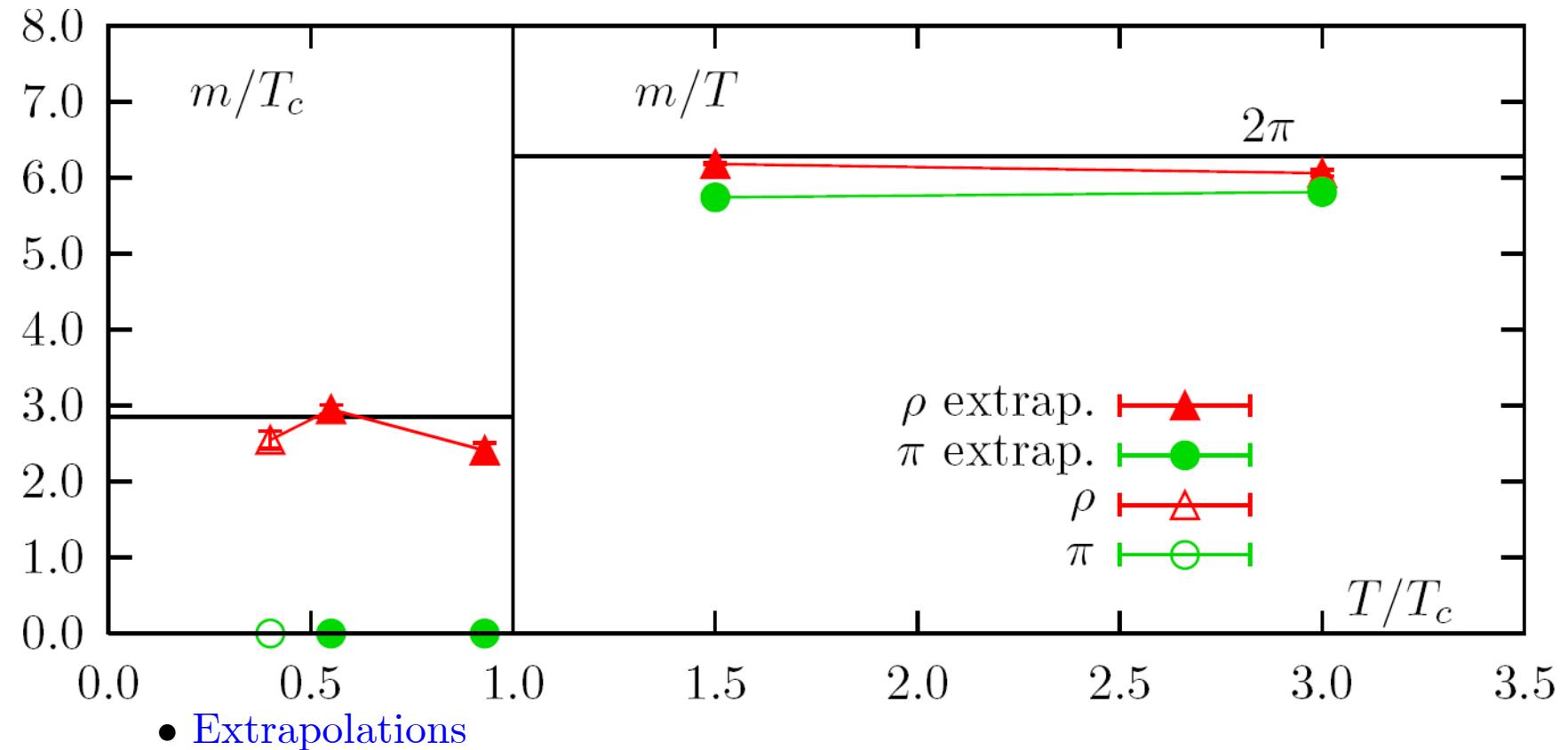


- Continuum extrapolation ansatz

$$m_{\text{screen}}(a) = m_{\text{screen}} - \lambda T(aT)^2 + \dots$$

- done for three values of $N_\tau=8,12,16$

Screening masses vs. temperature



- Extrapolations

- infinite volume done with ansatz obtained from $N_\tau=8$
- chiral limit (systematic error from determination of the critical hopping parameter)
- above T_c continuum extrapolation done with three points

Effective restoration of the $U_A(1)$

- Group-theoretical analysis
 - $N_f=3$ all $SU_L(3) \times SU_R(3)$ symmetric two-point functions of quark bilinears are automatically $U_A(1)$ symmetric
 - $N_f=2$ among $SU_L(2) \times SU_R(2)$ symmetric two-point functions there exist one $U_A(1)$ invariant and one $U_A(1)$ violating

M.C. Birse, T. D. Cohen, J. A. McGovern 1996

- Indications
 - Degeneracy of the pseudo-scalar and scalar correlators?
 - Contribution of configurations with non trivial topological charge?

Topology on lattice

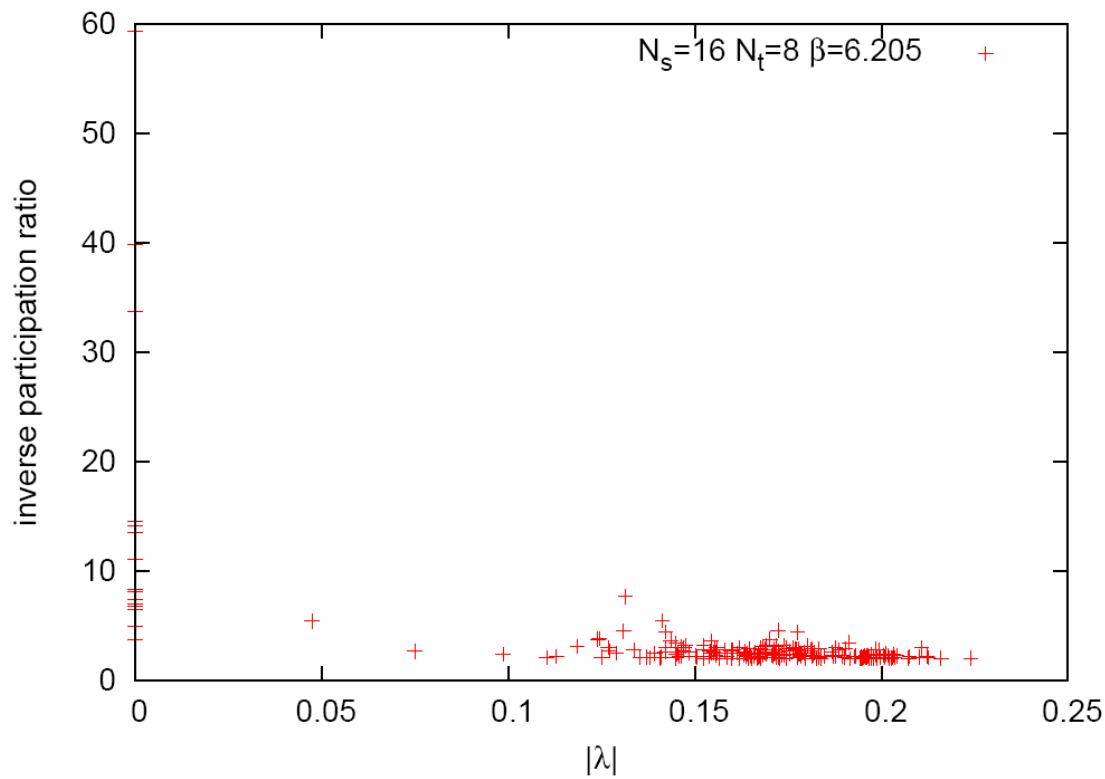
- Ginsparg-Wilson fermions
 - Exact lattice chiral symmetry M. Lüscher, 1998
 - No additive mass renormalization and operator mixing
 - Exact zero modes exist
 - Index defined through $N_+ - N_-$ P. Hasenfratz et al., 1998
- Solution: standard overlap operator (H. Neuberger, 1997)
$$D_{\text{ov}} = \frac{\mu}{a} \left(1 + \gamma_5 Q / \sqrt{Q^2} \right), \quad Q = \gamma_5 (a \widehat{D} - \mu), \quad \widehat{D} = D_w$$
- The overlap hypercube operator
 - \widehat{D} is the hypercube operator
 - Fat links $U_{x,\mu} \rightarrow (1 - \alpha)U_{x,\mu} + \frac{\alpha}{6} \sum [\text{staples}]$ W. Bietenholz 1998

Results for the restoration of the chiral symmetry

- $1.24T_C$ quenched
 - relatively frequent occurrence of configurations with $Q=1$ ($16^3 \times 8$ lattice)
 - zero modes and next-to-zero modes are highly localized
 - Pseudo-scalar and scalar correlators are not degenerate ($64^3 \times 16$ lattice)
 - Axial-vector and vector correlators are nearly degenerate → $SU(N_f) \times SU(N_f)$ restored ($64^3 \times 16$ lattice)
- $1.5T_C$ quenched
 - very few configurations with localized zero modes for $Q=1$ (seen only on $16^3 \times 8$ lattice)
 - Pseudo-scalar and scalar correlators are nearly degenerate ($64^3 \times 16$ lattice)
 - Axial-vector and vector correlators are nearly degenerate → $SU(N_f) \times SU(N_f)$ restored ($64^3 \times 16$ lattice)

Localization of the lowest eigenvectors at $1.24T_C$

Eigenvectors computed with the overlap operator



$$\text{inv.part.ratio} = V \sum_x (\psi^\dagger(x)\psi(x))^2$$

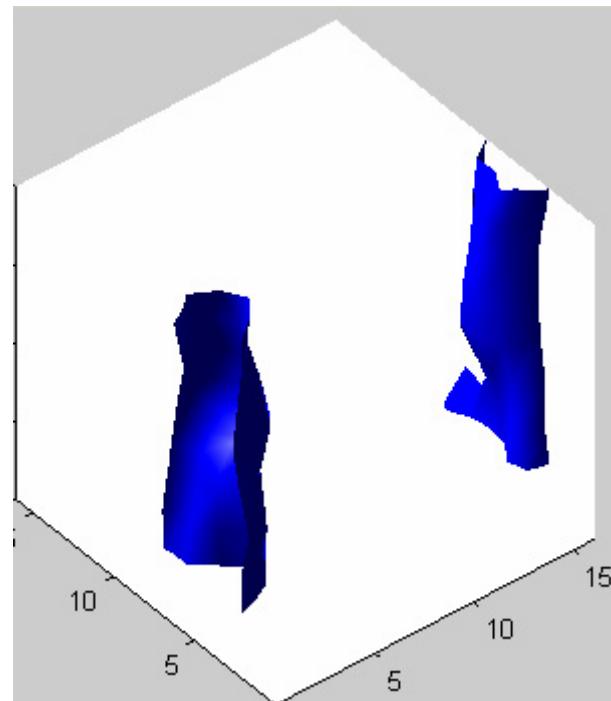
$$\begin{cases} \text{maximally localized} & V \\ \text{maximally spread out} & 1 \end{cases}$$

Localized zero mode and next-to zero mode in the deconfined phase

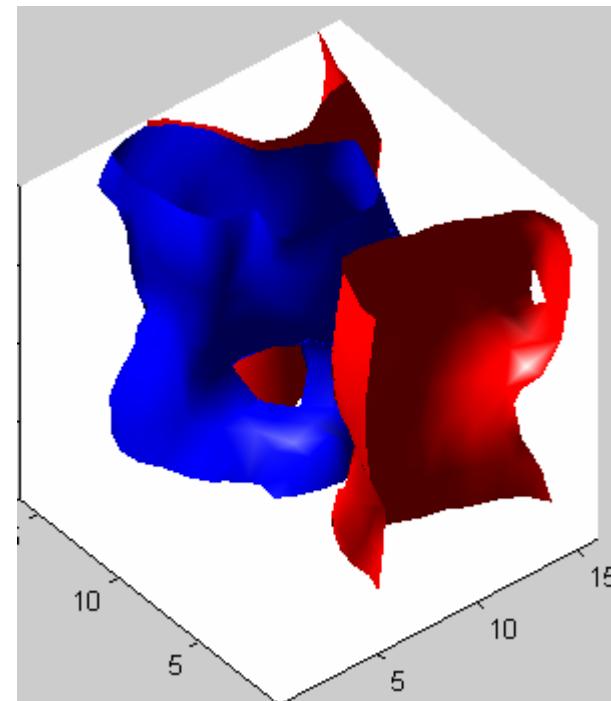
$V=16^3 \times 8$, isosurface $\psi^\dagger(x, y, z, \tau) \gamma_5 \psi(x, y, z, \tau) = const$, $z = const$

$$T = 1.5T_c$$

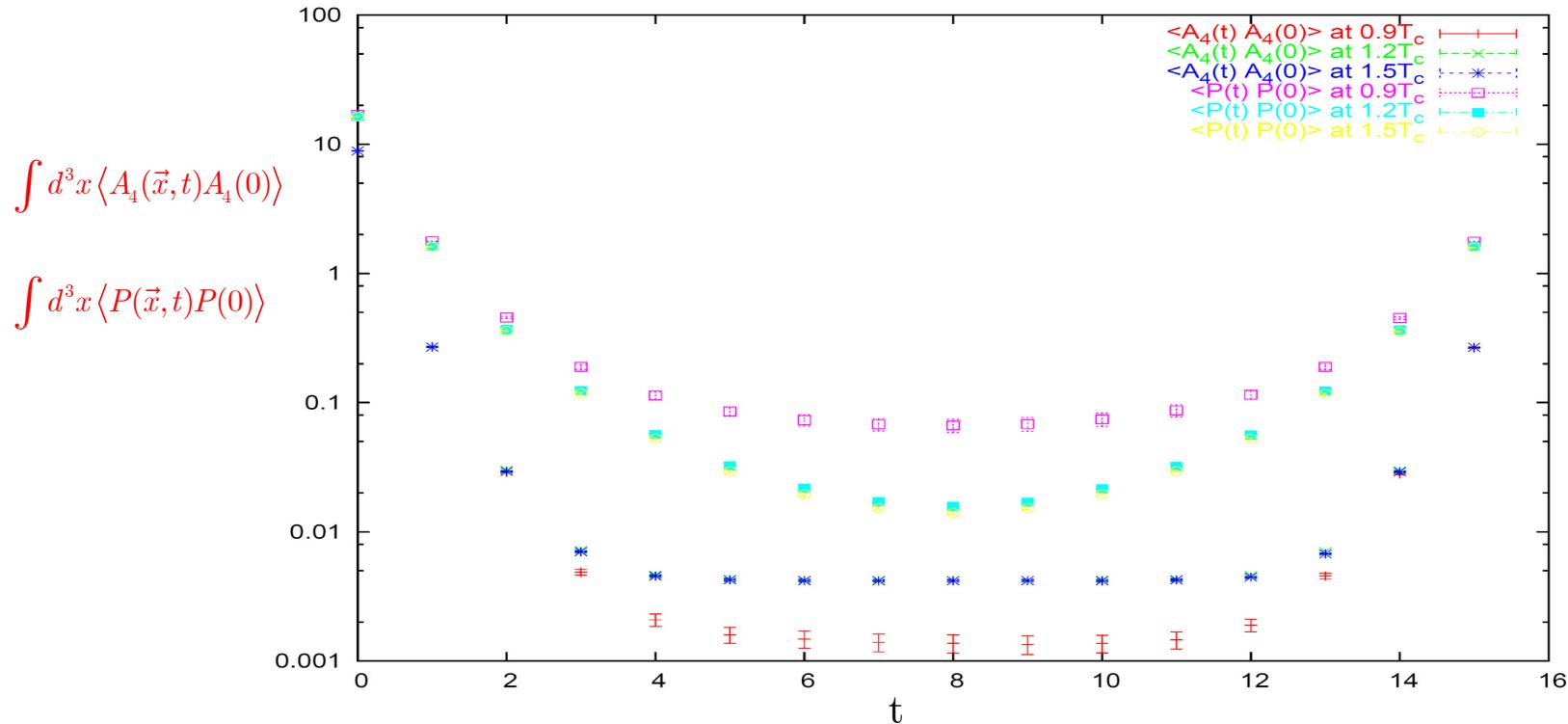
Zero mode



Next to zero mode



Restoration of $SU_L(N_f) \times SU_R(N_f)$ at $1.2 T_C$ and $1.5 T_C$ from temporal correlators



- The pseudoscalar correlator reveals non-plateau behaviour up to $1.5 T_C$
- The axial vector correlator $\langle A_4(t) A_4(0) \rangle$ is flat at $1.2 T_C$ and $1.5 T_C$

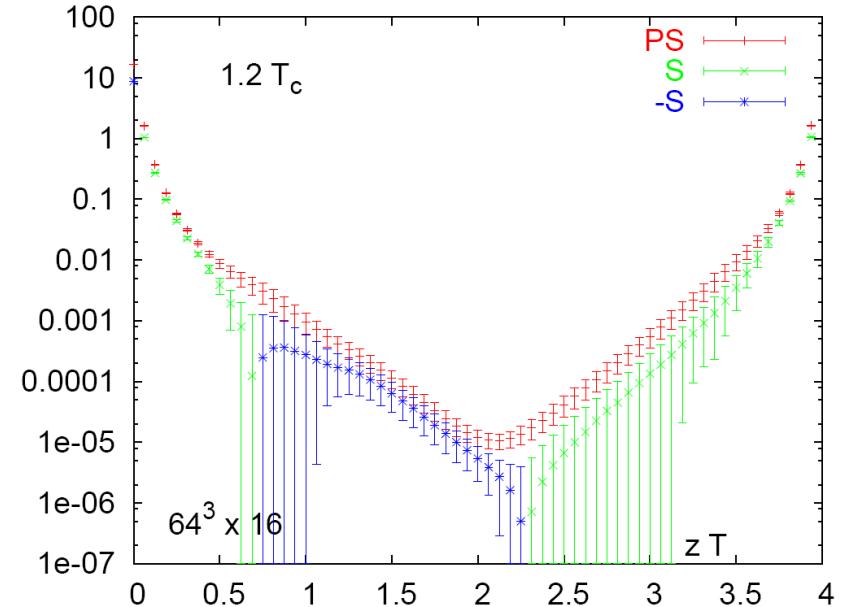
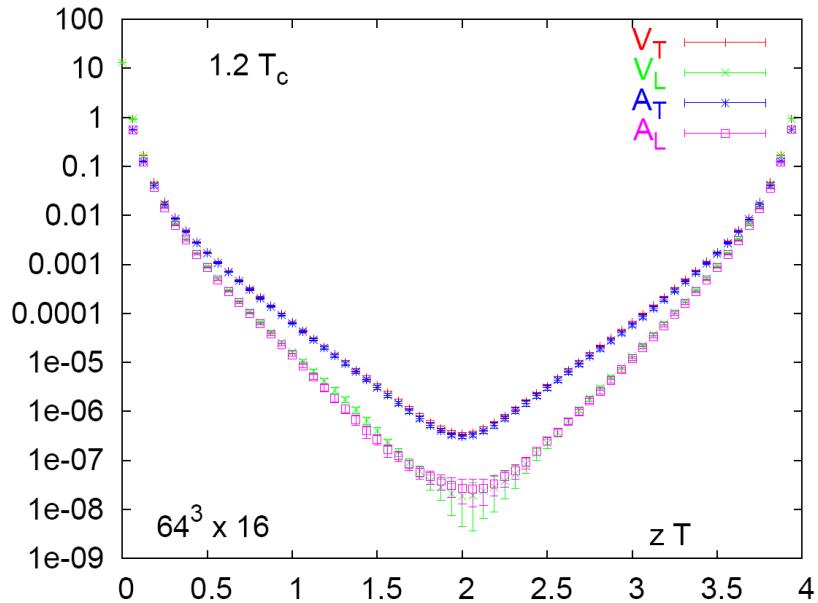
$$\int d^3x \langle A_4(\vec{x}, t) A_4(0) \rangle = \langle Q_A(t) A_4(0) \rangle = \text{const}$$

$$\rightarrow Q_A(t) = \text{const}$$

Results for the restoration of the chiral symmetry

- $1.24T_C$ quenched
 - relatively frequent occurrence of configurations with $Q=1$ ($16^3 \times 8$ lattice)
 - zero modes and next-to-zero modes are highly localized
 - Pseudo-scalar and scalar correlators are not degenerate ($64^3 \times 16$ lattice)
 - Axial-vector and vector correlators are nearly degenerate → $SU(N_f) \times SU(N_f)$ restored ($64^3 \times 16$ lattice)
- $1.5T_C$ quenched
 - very few configurations with localized zero modes for $Q=1$ (seen only on $16^3 \times 8$ lattice)
 - Pseudo-scalar and scalar correlators are nearly degenerate ($64^3 \times 16$ lattice)
 - Axial-vector and vector correlators are nearly degenerate → $SU(N_f) \times SU(N_f)$ restored ($64^3 \times 16$ lattice)

Restoration of the $U_A(1)$ and $SU_L(N_f) \times SU_R(N_f)$ at $1.2 T_C$



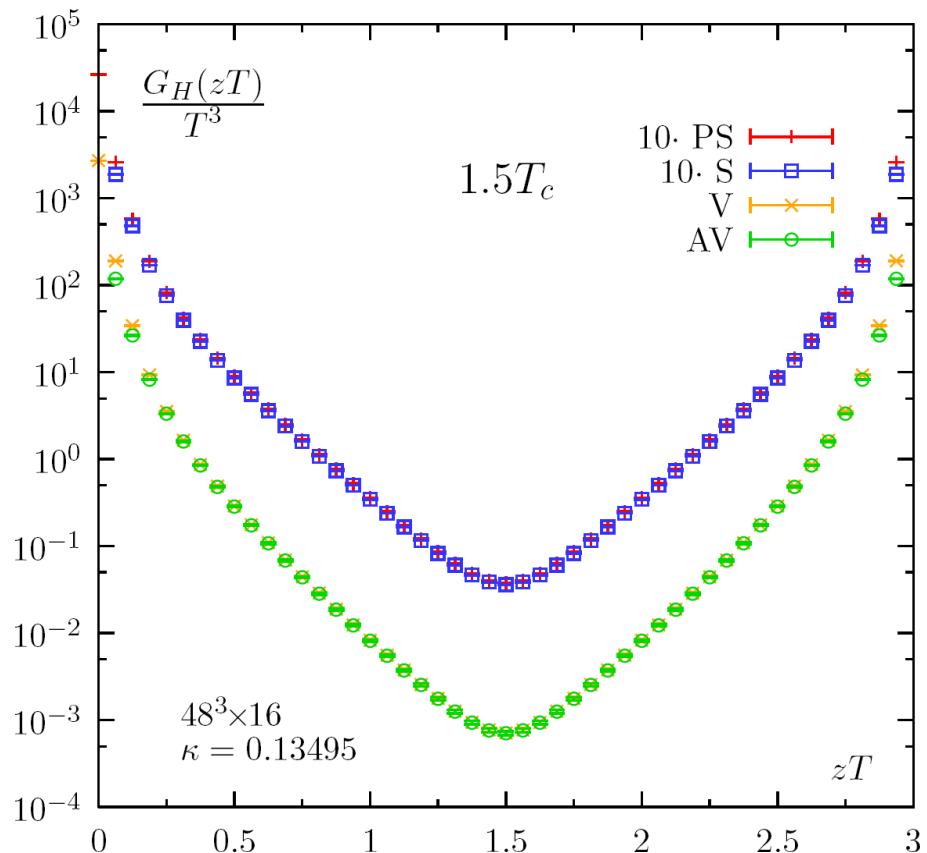
$1.2 T_C$: $V_T = A_T$,

$V_L = A_L$

but

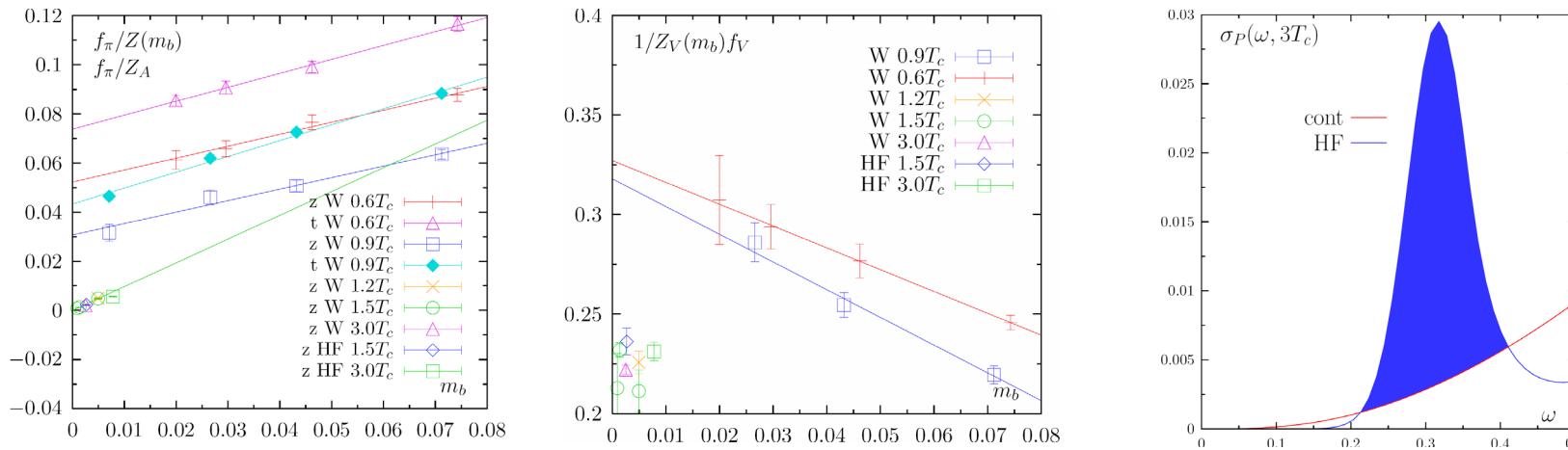
$PS \neq S$

Restoration of the $U_A(1)$ and $SU_L(N_f) \times SU_R(N_f)$ at $1.5T_c$



$1.5 T_c: V = A$ and $PS = S$

Decay constants at finite T



- Methods
 - From one cosh fit to temporal correlators ($\langle A_4 A_4 \rangle$)
 - From one cosh fit to spatial correlators ($\langle P P \rangle$)
 - From computation of the area under the peak in the spectral function
- Sources of errors
 - Difficult to find a plateau for the temporal correlators
 - MEM artifacts
 - confined phase: systematic error for the chiral extrapolation (the lowest pion mass is around 400 MeV)

Decay constants at finite T

- Results from spatial correlators

T/T _C	m (MeV)	f _P MeV	F _V MeV
0.55	0	135(5)	221(4)
0.93	0	137(8)	221(7)
1.2	37(12)	29(5)	370(10)
1.5	13(9)	9(6)	479(8)
3.0	46(12)	26(12)	989(21)

- Results from temporal correlators

T/T _C	m (MeV)	f _P MeV
0.55	0	142(2)
0.93	0	143(6)

- Results from spatial and temporal correlators roughly agree
- f_P goes to zero in the deconfined phase
- F_V increases in the deconfined phase

Conclusions

- The truncated perfect action might help to better disentangle physical information in the spectral functions from lattice artifacts than with the Wilson action
- The number of occurring excited states can be influenced by MEM artifacts
- The screening masses come close to the free value in the deconfined phase
- $U_A(1)$ symmetry might be effectively restored at $1.5T_C$ as quenched results with Wilson fermions suggest. The role of the zero modes still has to be clarified
- Pion decay constant goes fast to zero in the deconfined phase

Outlook

- Renormalisation constants and simulations at different lattice spacings are needed to interpret the “excited states”
- Results at larger spatial extensions needed to make a safe infinite volume extrapolation for the screening masses
- $N_f=2$ flavour simulations with good chiral properties are needed to address the violation of $U_A(1)$ symmetry in the two-point functions